EXPAND $(2x-y)^5 = (\mathbf{2x} + (\mathbf{-y}))^5$

LET $x = \mathbf{2x}$, $y = \mathbf{-y}$

$$= (2x)^5 + 5(2x)^4(-y) + 10(2x)^3(-y)^2 + 10(2x)^2(-y)^3 + 5(2x)(-y)^4 + (-y)^5$$

$$= 32x^5 + 5(16x^4)(-y) + 10(8x^3)(y^2) + 10(4x^2)(-y^3) + 5(2x)(y^4) + (-y^5)$$

$$= 32x^5 - 80x^4y + 80x^3y^2 - 40x^2y^3 + 10xy^4 - y^5$$

NOTE: ALTERNATING SIGNS +, −

EXPAND $\left(\sqrt{x} - \frac{2}{x}\right)^4 = (\mathbf{x^{\frac{1}{2}}} + (\mathbf{-2x^{-1}}))^4$

$(x^m)^n = x^{mn}$

$$= (x^{\frac{1}{2}})^4 + 4(x^{\frac{1}{2}})^3(-2x^{-1}) + 6(x^{\frac{1}{2}})^2(-2x^{-1})^2 + 4(x^{\frac{1}{2}})(-2x^{-1})^3 + (-2x^{-1})^4$$

$$= x^2 - 8x^{\frac{3}{2}}x^{-1} + 24xx^{-2} - 32x^{\frac{1}{2}}x^{-3} + 16x^{-4}$$

$$= x^2 - 8x^{\frac{1}{2}} + 24x^{-1} - 32x^{-\frac{5}{2}} + 16x^{-4}$$

SANITY CHECK:
IF EXPONENTS OF EACH VARIABLE do <u>NOT</u> FORM AN ARITHMETIC SEQUENCE, THEN THERE IS AN ERROR

NOTE: EXPONENTS OF X $2, \frac{1}{2}, -1, -\frac{5}{2}, -4$

$-\frac{3}{2}$ $-\frac{3}{2}$ $-\frac{3}{2}$ $-\frac{3}{2}$

FORM AN ARITHMETIC SEQUENCE

HOW TO FIND ENTRIES IN PASCAL'S TRIANGLE AT WILL (IE. ONLY 1 ENTRY)

ROW 0 → 1 ← COEF'S OF $(x+y)^0$

ROW 1 → 1 1 ← $(x+y)^1$

ROW 2 → 1 2 1 ← $(x+y)^2$

1 3 3 1

1 4 6 4 1

ENTRY 0, ENTRY 1, ENTRY 2

$$C(n,r) = \begin{matrix}\text{ROW } n\\ \text{ENTRY } r\end{matrix} = {}_nC_r = \binom{n}{r}$$

READ AS "n CHOOSE r"

$$C(3,2) = \begin{matrix}\text{ROW } 3\\ \text{ENTRY } 2\end{matrix} = 3$$

$$C(4,2) = 6 = \begin{matrix}\text{ROW } 4\\ \text{ENTRY } 2\end{matrix}$$

$$C(n,r) = \frac{n!}{r!\,(n-r)!}$$

$$C(4,3) = \frac{4!}{3!\,(4-3)!} = \frac{4!}{3!\,1!} = \frac{4\cdot\cancel{3!}}{\cancel{3!}\,1} = 4$$

$$C(7,3) = \frac{7!}{3!(7-3)!} = \frac{7!}{3!4!} = \frac{7\cdot\cancel{6}\cdot5\cdot\cancel{4!}}{\cancel{3\cdot2}\cdot1\cdot\cancel{4!}} = 7\cdot5 = 35$$

1							
1	1						
1	2	1					
1	3	3	1				
1	4	6	4	1			
1	5	10	10	5	1		
1	6	15	20	15	6	1	
1	7	21	$\underline{\underline{35}}$	35	21	7	1

ROW 4 $\longrightarrow$ 1 4 6 4 1

$$(x+y)^4 = C(4,0)x^4y^0 + C(4,1)x^3y^1 + C(4,2)x^2y^2 + C(4,3)x^1y^3 + C(4,4)x^0y^4$$

$$(x+y)^4 = \sum_{r=0}^{4} C(4,r)x^{4-r}y^r$$

$$\boxed{(x+y)^n = \sum_{r=0}^{n} C(n,r)x^{n-r}y^r}$$

BINOMIAL THEOREM

WHERE $n \in \mathbb{Z}^{\text{non-neg}}$ OR $\mathbb{Z}^*$

$$C(n,r) = \frac{n!}{r!(n-r)!}$$

$$(x+y)^n = \sum_{r=0}^{n} \binom{n}{r} x^{n-r} y^r$$

FIND THE FIRST 4 TERMS OF $(3x^4 - 2y^3)^{18} = (\underbrace{3x^4}_{x} + \underbrace{(-2y^3)}_{y})^{\overset{n}{18}}$

$$= \sum_{r=0}^{18} \binom{18}{r} (3x^4)^{18-r} (-2y^3)^r$$

$$= \sum_{r=0}^{18} (-1)^r \binom{18}{r} 3^{18-r} 2^r x^{4(18-r)} y^{3r}$$

FIRST 4 TERMS CORRESPOND TO $r = 0, 1, 2, 3$

$$= \binom{18}{0} 3^{18} x^{72} - \binom{18}{1} 3^{17} 2 x^{68} y^3 + \binom{18}{2} 3^{16} 2^2 x^{64} y^6 - \binom{18}{3} 3^{15} 2^3 x^{60} y^9 + \dots$$

$$= 3^{18} x^{72} - 18 \cdot 3^{17} \cdot 2 x^{68} y^3 + 9 \cdot 17 \cdot 3^{16} 2^2 x^{64} y^6 - 3 \cdot 17 \cdot 16 \cdot 3^{15} 2^3 x^{60} y^9 + \dots$$

$(-1)!$ DNE

$\div 0$

$0! = 1$ $\quad *1$

$\div 1$

$1! = 1$

$\div 2$ $\quad *2$

$2! = 2 \cdot 1$

$\div 3$ $\quad *3$

$3! = 3 \cdot 2 \cdot 1$

$\div 4$ $\quad *4$

$4! = 4 \cdot 3 \cdot 2 \cdot 1$

$$\binom{n}{0} = \frac{n!}{0!(n-0)!} = \frac{\cancel{n!}}{1 \cdot \cancel{n!}} = 1$$

$$\binom{n}{1} = \frac{n!}{1!(n-1)!} = \frac{n\cancel{(n-1)!}}{1 \cdot \cancel{(n-1)!}} = n$$

$$\binom{18}{2} = \frac{18!}{2!\,16!} = \frac{\overset{9}{\cancel{18}} \cdot 17 \cdot \cancel{16!}}{\cancel{2} \cdot 1 \cdot \cancel{16!}} = 9 \cdot 17$$

$$\binom{18}{3} = \frac{18!}{3!\,15!} = \frac{\overset{3}{\cancel{18}} \cdot 17 \cdot 16 \cdot \cancel{15!}}{\cancel{3 \cdot 2} \cdot 1 \cdot \cancel{15!}} = 3 \cdot 17 \cdot 16$$

FIND THE 19^{TH} TERM OF $\left(\frac{3}{x} - 2\sqrt{x}\right)^{23} = \Big(\underbrace{3x^{-1}}_{x} + \underbrace{(-2x^{\frac{1}{2}})}_{y}\Big)^{23} \leftarrow n$

$r = 18$

$$= \sum_{r=0}^{23} \binom{23}{r} (3x^{-1})^{23-r} (-2x^{\frac{1}{2}})^{r}$$

$$= \sum_{r=0}^{23} (-1)^r \binom{23}{r} 3^{23-r} 2^r x^{r-23+\frac{1}{2}r}$$

$$= \sum_{r=0}^{23} (-1)^r \binom{23}{r} 3^{23-r} 2^r x^{\frac{3}{2}r-23}$$

$$19^{TH} \text{ TERM} = (-1)^{18} \binom{23}{18} 3^{23-18} 2^{18} x^{\frac{3}{2}(18)-23}$$

$$= 23 \cdot 11 \cdot 7 \cdot 19 \cdot 3^5 2^{18} x^4$$

$$\binom{23}{18} = \frac{23!}{18!\,5!} = \frac{23 \cdot \overset{11}{\cancel{22}} \cdot \overset{7}{\cancel{21}} \cdot \cancel{20} \cdot 19 \cdot \cancel{18!}}{\cancel{18!}\ \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1}$$